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THE MATHEMATICS TEACHER

VOLUME XVII NOVEMBER, 1924 NUMBER 7

THE SLIDE RULE IN PLANE GEOMETRY [1]

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The first place where the Slide Rule has a real application in Plane Geometry is in the study of proportion and similar figures in Books III, IV and V. After the proposition is taken up in the class in the usual way, the numbers for the various known lines in a figure are assigned and the unknown lines are computed. Very often the result is found by one setting of the slide by reading the numbers under the runner. In the case a continued proportion, any number of answers may be read off by reading them under the runner.

Proportion

A proportion may be solved on either the *A* and *B* scales or *C* and *D* scales. Many prefer to do this on the *A* and *B* scales, as one will never need to shift the slide, as it sometimes occurs on the *C* and *D* scales.

The explanation which follows will be carried out on the *C* and *D* scales, but the remarks hold equally well when the *A* and *B* scales are used.

Place the left index of *C* scale over 2 of *D* scale, and you will notice that 2, 3, 4, 5 of the *C* scale, are opposite 4, 6, 8 and 10 of the *D* scale, thus forming the continued proportion, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10}$. If the right index of *C* scale is placed over 2 of *D* scale, then under 6, 7, 8, 9, 10 of the *C* scale will be found 12, 14, 16, 18, 20 of the *D* scale respectively.

Example 1. $\dfrac{14.2}{22.3} = \dfrac{56.8}{X}$. Find X.

Following is the diagramatic setting for the above example:

$$\frac{C \quad 14.2 \quad 56.8}{D \quad 22.3 \quad X = 89.2}$$

Place 14.2 of *C* scale above 22.3 of *D* scale, then under 56.8 of *C* scale will be found the answer $x = 89.2$ on *D* scale. The

[1] Part of a Report of a Committee of the Chicago Men's Mathematical Club.

decimal point is easily placed by noticing that 14.2 is about ⅔ of 22.3 and 56.8 will be, therefore, ⅔ of the answer X.

The rule for proportion is, therefore, to place the 1st and 3rd terms on the C scale and the 2nd and 4th will be found on the D scale directly below.

Example 2. $\frac{7.35}{36.6} = \frac{3.25}{X}$; $\begin{array}{l|lll} C & 7.35 & & 3.25 \\ \hline D & 36.6 & & X = 16.2 \end{array}$

Example 3. $\frac{52.7}{14.8} = \frac{X}{8.3}$; $\begin{array}{l|lllll} C & 52.7 & R \text{ to } 1 & 1 \text{ to } R & & X = 29.6 \\ \hline D & 14.8 & & & & R \text{ to } 8.3 \end{array}$

Example 4. $\frac{1.26}{X} = \frac{3.82}{27.2}$; $\begin{array}{l|ll} C & 3.82 & 1.26 \\ \hline D & 27.2 & X \end{array}$

This problem and the one before, you will find cannot be done in the way suggested, as 1.26 on C is too far to the left, and there is no D scale below it. Whenever this occurs, the runner should be moved to the right until it passes through the right C index, and then move the slide until it projects to the right and the left C index comes to the runner. This is called shifting or reversing the slide. Now under 1.26 answer will be found on the D scale.

C	3.82	Runner to R .1	to R	1.26
D	27.2			$X = 8.97$

Continued Proportion

Example 5. $\frac{3.62}{16.7} = \frac{4.45}{x} = \frac{5.79}{y} = \frac{6.65}{z} = \frac{1.20}{w}$

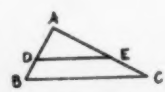
When 3.62 on C is placed about 16.7 of D scale, x, y, z may be read off without any additional movement of the slide.

The above examples should be done also on the A and B scales. You will notice you will never have to shift the slide.

Theorem 1. *A parallel to one side of a triangle, intersecting the other two sides, divides the other two sides proportionately.*

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{or } AB : AC = AD : AE$$

Example 6. $AB = 12.3$
$AC = 64.8$
$AD = 7.9$
Find AE



(Fig. 1)

C	12.3	R to 1	1 to R	7.9
D	64.8			$41.6 = AE$

$AD = 23.8$ $\quad AC = 97$
$DB = 57.9$ $\quad$ Find AE & EC
$AB = 81.7$

C	81.7	97
D	23.8	$AE = 28.2$
		$EC = 68.8$

$AD = 6.42$

$AD = 6.42$ $\quad AC = 12.45$
$DB = 3.38$ $\quad$ Find AE & EC

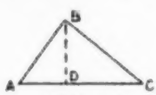
Here the slide rule will work out the two segments AE and EC directly.

C	6.62	$8.15 = AE$
D	3.38	$4.3 = EC$
		Check 12.45

After 6.42 is placed on C above 3.38 on D, look along the C & D until two numbers are found whose sum is 12.45. 8 on C scale is close to 4 on D and the sum of these numbers is 12. So it is easy to find any two numbers in a given ratio and equal to a certain sum.

Theorem 2. In any triangle, the bisector of an interior angle divides the opposite side internally into segments proportional to the adjacent sides of the triangle. $AD : DC = AB : BC$.

Example 8. $AB = 6.8$
$BC = 9.3$
$AC = 8.5$



(Fig. 2)

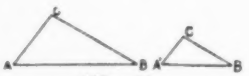
If the three sides of the triangle ABC are given as above, the segments BD & DC may be found in the following manner:

C	6.8	$AD = 3.59$
D	9.3	$DC = 4.91$
		Check $AC = 8.59$

As in the above example, when 6.8 is placed above 9.3 on C and D scales, you must look along these scales until two numbers are found whose sum is 8.5.

Theorem 3. *Two triangles that are similar have their corresponding sides proportional.*

$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$



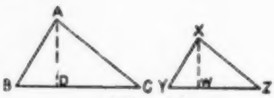
(Fig. 3)

Example 9. $AB = 3.68$ $A'B' = 2.75$
$AC = 5.84$ Find AC & $B'C'$
$BC = 4.72$

C	3.68	5.84	4.72
D	2.75	$A'C' = 4.37$	$B'C' = 3.53$

Theorem 4. *Homologous altitudes of similar triangles have the same ratio as any homologous sides.*

$$\frac{BC}{yz} = \frac{AD}{xw}$$



(Fig. 4.)

Example 10. $BC = 12.3$
$yz = 8.72$
$AD = 6.42$
Find xw

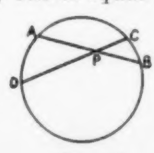
C 12.3	R to 1	1 to R	6.42	
D 8.72			$XW = 4.55$	

Example 11. $BC = 6.42$ $xw = 3.42$
Area $BAC = 12.6$ Find yz
$12.6 = \frac{1}{2}BC.\ AD.$ $AD = \frac{2x\ 12.6}{BC} = \frac{25.2}{6.42}$

C	6.42	1	R to 1	3.42	$yz = 5.59$
D	25.2	AD		AD	6.52

Theorem 5. *If two chords are drawn through a fixed point within a circle, the product of the segments of one is equal to the product of the segments of the other.*

$$AP \times PB = DP \times PC$$



(Fig. 5)

Example 12. $AP = 3.42$
$PB = 4.36$ Find PC
$DP = 6.18$

$$PC = \frac{3.42 \times 4.36}{6.18} \quad \text{or} \quad \frac{6.18}{3.42} = \frac{4.36}{PC}$$

C	6.18	4.36
D	3.42	$PC = 2.41$

Example 13. Find the segments of *AB* if the product of the segments of *DC* is 144 and their sum is 30.

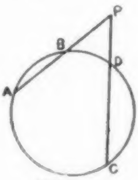
CI	1	6
D	144	24

Sum 30

By setting 144 on *D* to 1 on *CI* scale, the product of any two opposite numbers on *CI* & *D* will be 144. It remains then to select the two numbers whose sum is 30.

Theorem 6. *If any two secants are drawn through a fixed point outside a circle, the product of one and its external segment equals the product of the other and its external segment.*

$$AP \times PB = DP \times CP \text{ or } \frac{AP}{CP} = \frac{DP}{PB}$$



(Fig. 6)

Example 14. $AP = 12.4$, $BP = 6.35$, $DP = 3.2$ Find *CP*.

C	3.2	12.4
D	6.35	$CP = 24.6$

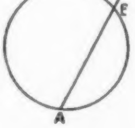
Theorem 7. *A tangent to a circle from an external point is the mean proportional to the whole secant and its external segment.*

$$CP^2 = AP \times PB$$

Example 15. $CP = 4.65$ $AP = 2.48$ Find *PB*

A	CP^2	*PB*	8.7
B		2.48	
C			
D	4.65		

$$\frac{4.65^2}{2.48} = X. \quad PB$$



(Fig. 7)

When a number is placed on a *D* scale, its square is on *A* scale above it.

Example 16. $AP = 2.78$ Find CP $CP^2 = 2.78 \times 1.46$
$PB = 1.46$

A	2.78	CP^2	
B	1	1.46	
C			
D		CP	Ans. $= 2.02$

Example 17. CP — 5.45, and the sum of PB and AP is 11.3.

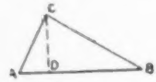
A	CP	$7.2 = AP^2$	
C			
$B1$		$4.1 = PB$	$7.2 - 4.1 = 3.1 =$ .AB
D	5.45		
		Sum 11.3	

Example 18. $CP = 4.93$ and the sum of AB and BP is 10.2.

A	CP^2	6.4	$= BP$	
$C1$				$6.4 = 3.8 = 2.6 = PB$
$B1$		3.8	$= AB$	
D	4.93			$4.93 = APXPB$
			Sum 10.2	10.2

Theorem 8. *If the altitude be drawn to the hypotenuse of a right triangle TBC, then*

$$CD^2 = AD.DB$$
$$CA^2 = AD.AB$$
$$CB^2 = AB.DB$$



(Fig. 8)

Example 19. $AD = 4.52$ Find CD,
$DB = 5.38$

$$CD = \sqrt{4.62 \times 5.38}$$

A	4.62	CD^2	
B	z	5.38	
C			
D		$CD = 4.98$	

Example 20. $CD = 7.86$
$AD = 4.32$
Find DB

$$DB \quad \frac{CD^2}{AD} = \frac{7.86^2}{4.32}$$

A	CD^2	$DB = 14.3$	Ans.
B	4.32	1	
C			
D	7.86		

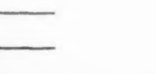
Example 21. $CD = 1.89$
$AB = 3.85$ Find AD & DB

A	1.89	$1.6 = AB$
CI		
BI	1	$2.25 = AD$
D	1.89	

Check 3.85

To get BI scale pull the slide out and turn it end for end. Find the number 1.6 and 2.25 on A & BI scales, so that their sum is 3.85

Theorem 9. *The sum of the squares of the two legs of a right triangle equals the square of the hypotenuse.*



(Fig. 9)

$$AB^2 + BC^2 = AC^2$$

Example 22. $AB = 5.62$, $BC = 12.35$ Find AC

A	$31.5 + 152.5$	184.0	left half A
B			
C			
D	5.62	12.35	$13.58 = AC$

Example 23. $BC = 8.32$ $AC = 14.96$ Find AB

$$AB = \sqrt{14.96^2 - 8.32^2} = \sqrt{(14.96 - 8.32)(14.96 + 8.32)}$$
$$= \sqrt{6.64 \times 25.28}$$

A	6.64	AB^2
B	1	23.28
C		
D		12.4 Ans

AB is a number having three digits; starting this multiplication on the left *A* brings its result on the right *A*. In order to find its square root it will have to be transferred to left *A*. To do this bring middle *B* index to product and under left index *B* on *D* the result will be found.

Example 24.

$AB = 5 \quad BC = 12$ and $AC = 13$

Find AD, DC and BD.

A	12^2	$11.1 = AD$
B	13	1
C		
D	12	

$$AB^2 = AC \times AD$$

$$\frac{12}{13} = AD$$

$$CB^2 = AC.\ DC$$

$$\frac{5^2}{13} = DC$$

A	52	$1.92 = DC$
B	13	1
C		
D	5	

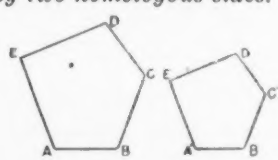
13.02 = AC

A	11.1	
B	1	1.92
C		
D		$4.63 = BD$

$$BD = AD.\ DC$$

$$BD = \sqrt{11.1 \times 1.92}$$

Theorem. 10. *The perimeters of two similar polygons have the same ratio as any two homologous sides.*



(Fig. 10)

$$\frac{AB + BC + CD + DE + EA}{A'B' + B'C' + C'D' + D'E' + E'A'} = \frac{AB}{A'B'} \text{ etc.}$$

Example 25.

$AB = 3.62$ Perimeter $A'B'C'D'E' = 12.32$
$BC = 4.31$
$CD = 1.68$ Find: $B'C'$
$DE = 2.45$ $C'D'$
$EA = 5.84$ $D'E'$

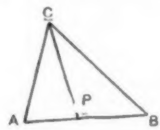
Perimeter = 17.90 $E'A'$
$A'B'$

$$\frac{17.90}{12.32} = \frac{3.62}{A'B'} = \frac{4.31}{B'C'} = \frac{1.68}{D'E'} = \frac{2.45}{D'E'} = \frac{5.84}{E'A'}$$

A	17.90	3.62	4.31	2.45	5.84	1.68
B	12.32	$A'B' = 2.48$	$B'C' = 2.97$	$D'E' = 1.68$	$E'A' = 4.02$	$D'E' = 1.16$

***Check*:** $2.48 + 2.97 + 1.68 + 1.16 = 12.31$

Theorem 11. *The square of the bisector of an angle of a triangle is equal to the product of the sides of this angle diminished by the product of the segments made by the bisector upon the third side of the triangle.*



(Fig. 11)

$CP^2 = AC \times BC - AP \times PB$

Example 26. $AC = 34.3$
$AB = 22.8$
$CB = 17.8$
Find CP

Since CP bisects Angle C $\dfrac{AP}{PB} = \dfrac{AC}{CB}$

C	34.3	R to 1	1 to R	15.02
D	17.8			7.78

$AB = 22.8$ Check

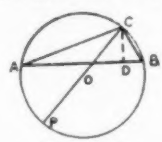
$CP^2 = 34.3 \times 17.8 - 15 \times 7.78$

A	594
B	
C	$24.4 = CP$

C	1	17.8	1	15
D	34.3	611	7.78	117

$611 - 117 = 594$

Theorem 12. *In any triangle the product of two sides is equal to the product of the diameter of the circumscribed circle by the altitude upon the third side.*



(Fig. 12)

$CA \times BC = DC \times CP$

Or $\dfrac{CP}{BC} = \dfrac{CA}{CD}$

Example 27. $AC = 35.6$ $CP = 13.8$
$CB = 27.3$ Required CD the diameter

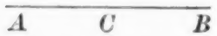
C	13.8	35.6
D	27.3	$70.3 = CD$

Extreme and Mean Ratio. **(Definition.)**

If a line is divided into two segments such that one segment is the mean proportional between the whole line and the other segment, the line is said to be divided in extreme and mean ratio.

Example 28.

$$\frac{AB}{AC}=\frac{AC}{CB} \qquad AB = 22''$$



Required segments AC and CB.

$$AC^2 = AB \times CB$$

A	22^2	$13.6 = AC$	Ans.
CI			
BI		35.6	
D	22		

Check $35.6 - 13.6 = 22$

Let $AC = X$

$X^2 = (22X)22$

$X^2 + 22X - 484 = 0$

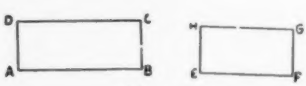
Example 29. $AB = 26.8$; to find AC.

A	26.8^2	16.6
CI		
BI		43.4
D	26.8	

Check $43.4 - 16.6 = 26.8$

The above problem illustrates how any quadratic equation may be solved.

Theorem 13. *Two rectangles having equal altitude are to each other as their bases.*



(Fig. 13)

$$\frac{ABCD}{HEFG}=\frac{AB}{EF}$$

C	283	24	
D	176	14.93	EF Ans.

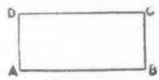
Example 30. $ABCD = 283$
$AEDG = 176$
$AB = 24$ Find AE

Example 31. $ABCD = 245$
$AEFG = 135$
$AB + AE = 63.6$ Find $AX \times AE$

C	245	$41 = AB$
D	135	$22.6 = AS$ Ans.

Check 63.6

Theorem 14. *The area of a rectangle equals the product of its base and altitude.*



(Fig. 14)

Area $ABCD = BC \times AB$

Example 32.

$AB = 12.3$

$BC = 37.8$

Find Area

C	1	37.8	
D	12.3	465	Ans.

Example 33.

Area $= 265$

$BC = 126$ Find AB

C	126	1	
D	265	$2.1 = AB$	Ans.

Example 34.

Area $= 468$

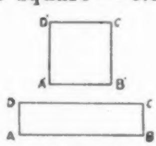
Sum of AB and $BC = 44$

CI	1	$25.8 = BC$	Ans.
D	468	$18.2 = AB$	
	Check	44.0	

Example 35.

Rectangle $ABCD =$ area of square $A'B'C'D'$

$AB = 12.6$ side of square $= 6.8$ Find AD



(Fig. 15)

$$AD \times AB = A'B'^2$$

$$AD = \frac{6.8^2}{12.6}$$

A	6.8^2	6.8^2	$3.67 = AB$	Ans.
B		12.6	1	
C				
D	6.8			

Example 36.

Rectangle $ABCD =$ square $A'B'C'D'$, $A'B' = 7.8$, sum of base and altitude $ABCD = 5.61$. Find AB and AD.

Example 37.

A	7.8^2	7.8^2	$1.46 = AD$
CI			
BI		1	$4.15 = AB$
D	7.8		
		Check	5.61

***Example* 38.** Rectangle $ABCD$ = square $A'B'C'D'$

$A'B' = \frac{8.32}{2}$ Diagonal of square $A'C' = 8.32$

Difference between altitude and base of rectangle = 3.27

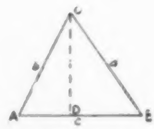
Find AB & BC.

A	8.32^2	area of sq.		area of sq.	8:40 = AB
B	2	1	*CI*		
C			*BI*	1	4.13 = BC
D	8.32				

Check 3.27

Theorem 14. *Area of a triangle equals the product of one half of the base by its altitude.*

$$\text{Area } ABC = \frac{AB \times CD}{2}$$



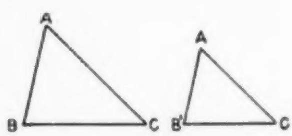
(Fig. 16)

Example 39. $AB = 36.9$
$CD = 12.8$
Find area ABC

C	1	36.9	2 to *R*	1
D	12.8			236 Ans

Theorem 15. *The area of two similar triangles are to each other as the squares of two corresponding sides.*

$$\frac{\text{Triangle } ABC}{\text{Triangle } A'B'C'} = \frac{AB^2}{A'B'^2}$$



(Fig. 17)

***Example* 40.** $AB = 36.8$
$A'B' = 17.3$
Area $ABC = 148$

$$\frac{A'B'C'}{148} = \frac{(17.3)^2}{(36.8)}$$

A			32.7 = $A'B'C'$ Ans.
B			148
C	36.8	1	
D	17.3		

Example 41. Area $ABC = 326$
Area $A'B'C' = 178$
$A'B' = 18.3$ Find AB

$$\frac{AB}{18.3^2} = \frac{236}{178} \qquad AB = 18.3\sqrt{\frac{236}{178}}$$

A	236	1.32			
B	178	1			
C			1	183	
D		1.15	1.15	21.0 — *AB*	Ans.

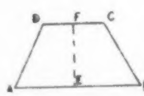
Example 42. *Area* $ABC = 128$
Base $AB = 14.7$
Find altitude CD

$$CD = \frac{128 \times 2}{14.7}$$

C	14.7	2	
D	128	17.4 = *CD*	Ans.

Theorem 16. *The area of a trapezoid is equal to half the product of the sum of its bases and altitude.*

$$\text{Area } ABCD — \frac{EF}{2} \quad (AB \times CD)$$



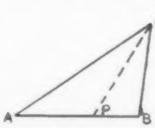
(Fig. 18)

Example 43. Area — 176
Bases = 14.7 & 6.35
Find altitude

C	21.05	2	
D	176	16.7 = *EF*	Ans.

Theorem 17. *In any triangle the square on the side opposite an acute angle is equivalent to the sum of the squares on the other two sides diminished by twice the product of one of those sides by the projection of the other upon that side.*

$$a^2 = b^2 + c^2 - 2bc \times AD$$



(Fig. 19)

Example 44. $b = 28.6$ $c = 18.3$ $AD = 7.38$
Find a

$$a^2 = 28.6^2 + 18.3^2 - 2 \times 28.6 \times 18.3 \times 7.38$$

A	$b^2 + c^2$	2						
B	818 + 335 — 1153	1	28.6	1 to *R*	18.3	1 to *R*	7.38	1065.3
C								
D	28.6 18.3						87.7	32.6
								Ans.

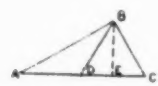
$$1153 - 87.7 = 1065.3$$

$$\sqrt{1065.3} = 32.6 \quad \text{Ans.}$$

Theorem. 18. *The sum of the squares of two sides of a triangle is equivalent to twice the square of half the third side increased by twice the square on the median upon that side.*

The differences of the squares on two sides of a triangle is equivalent to twice the product of the third side by the projection of the median upon that side.

$AB^2 + BC^2 \qquad AD^2 + 2BD^2$



(Fig. 20)

Example 45. $AB = 34.6 \qquad BC - 18.7$

$AC = 24.2 \qquad AB = 12.1$

Find BD the median, DE the projection on the median, BE the altitude and the area of ABC.

$$\sqrt{\frac{34.6^2 + 18.7^2}{2} - 2 \times 12.1^2 - BD}$$

A	1200 + 350 — 293 — 1257					
B			2	2	1	
C						
D	34.6	18.7	12.1			24.9 = BD

$$AB^2 - BC^2 = 2 \times AC.DE$$

$$\frac{(AB \times BC) \quad (AB - BC)}{2\ AC} = DE$$

$$\frac{(34.6 \times 18.7) \quad (34.6 - 18.7)}{2 \times 24.2} = DE$$

C	2	15.9	slide to 24.2	1
D	53.3			17.5 — E*D*

$= \sqrt{BD^2 - DE^2} = BE$

$= \sqrt{(24.9 - 17.5)\quad(24.9 \times 17.5)} = BE$

$\sqrt{42.4 \times 7.4}$

A	42.4				
B	1	7.4	1 to *R*	*I*	
C				17.72	*BE* altitude of triangle *ABC*
D					

Area $ABC = \dfrac{17.72 \times 24.2}{2}$

C	1	17.72	
D	12.1	21.5	Ans = Area

Theorem 19.

Area of triangle $ABC - /\overline{s\ (s-a)\ (s-b)\ (s-c)}$

Where $s = \dfrac{a+b+c}{2}$

$AB = 34.6$
$BC = 18.7$ Same numbers as above problem.
$AC = 24.2$

$2\,/\,77.5$
$s = 38.75$ Area $ABC\ /\overline{38.75 \times 4.15 \times 20.05 \times 14.55}$

$s - a = 4.15$
$s - b = 20.05$
$s - c = 14.55$

A	38.75					
B	1	42.9	1 to *R*	20.05	1 to *R*	14.55
C						
D						21.60

Theorem 20. *The circumference of any circle is equal to the diameter multiplated by* π.

$C - 2\,\pi \qquad R = \pi\, D$

Example 46. $R = 3.43$; find *C*.

C	1	6.38	
D	3.43	21.4	Ans.

$\dfrac{C}{D} = \dfrac{3.1416}{1} = \dfrac{710}{226}$

3.1416 is too hard to set off on the C or D scale and do it accurately; $^{710}/_{226}$ gives 3.1416 very accurately and these numbers are very easily set off.

Example 47. Given $D = 2.38$ Find circumference.

C	710	7.48 Ans
D	226	2.38

Example 48. $D = 4.62$. Find C.

C	710	R to 1	1 to R	14.5 Ans.
D	226			4.62

Example 49. $C = 24.8$. Find D.

C	710	R to 1	1 to R	24.8
D	226			7.9 Ans.

Theorem 21. *The area of a regular polygon is equal to half the product of its apothem by the perimeter.*

$$s = \tfrac{1}{2}\, a\text{-}p$$

Example 50. $P = 243$ $A = 863$. Find S.

C	1	8.63	2 to R	1
D	243			1050 Ans.

Example 50-A. Permieter = 465, area = 1648, find apothem.

$$A = \frac{2s}{P}$$

C	465	R to 1	1 to R	1648
D	2			706 Ans.

Theorem 22. *The area of a circle is equal to half the product of its radius by its circumference.*

$$A = \pi R^2 = \frac{\pi D^2}{4} = 3.1416R^2 - .7854D^2$$

Example 51. $R = 3.46$. Find A.

A		27.7	Ans.
B	1	π	special mark
C			
D	3.46		

Example 52. $D = 8.42$. Find A.

A			55.5	Ans.
B		.7854 —	π	special mark
C	1			
D	8.42			

Example 53. $A = 54.5$. Find R.

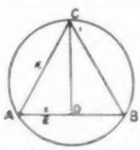
A	54.4	
B	π	1 middle index
C		
D		4.17

Example 54. $A = 425$. Find D.

A	425	
B	.7854	1 middle B
C		
D		23.3 Ans.

Example 55. Given $R — 4.72$. Find the side and area of inscribed regular triangle.

$$CD = 4.72$$
$$\underline{2.36}$$
$$= 7.08$$



(Fig. 21)

$$AC^2 - CD^2 = AD^2 \text{ or } X^2 = 7.08^2 + \frac{(X^2)}{2}$$

$$\frac{3}{4} \times 2 = 7.08^2, \quad x^2 = \frac{4 \times 7.08^2}{3}$$

$$X = \frac{2 \times 7.08}{3} \sqrt{3}$$

A				
B				3
C	3	7.08	1 to R	
D	2			8.18 =

Perimeter = 24.54

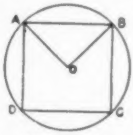
$$\text{Area} = \frac{24.54}{2} \times 2.35 = 12.27 \times 2.36$$

C	1	2.36
D	12.27	289

Example 56. Given $R = 4.72$. Find the side and area of the inscribed square.

$AB^2 = AO^2 + OB^2 = 2AO^2$

$AB = 4.72\sqrt{2}$



(Fig. 22)

A			
B		2	
C	1		
D	4.72	6.74	*AB*

Area $= AB^2 = 2 \times 4.72^2$

A	4.72^2	45	Ans.
B	1	2	
C			
D	4.72		

$$\frac{\text{Diameter of circle}}{\text{Side of inscribed square}} = \frac{99}{70} = \frac{\sqrt{2}}{1}$$

C	70	*R* to 1	1 to *R*	6.74	Ans.	checks
D	99					ans. above

$$\frac{\text{Area of circle}}{\text{Area of inscribed square}} = \frac{322}{205} = \frac{1.57}{1}$$

$$\frac{\text{Area of circle}}{\text{Area of inscribed square}} = \frac{\pi R^2}{2R^2} = \frac{\pi}{2} = 1.57$$

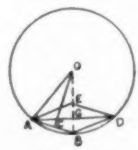
Area of circle R, $4.72 = 70$

C	322	70	
D	205	45.2	area checks above

Example 57. $R = 4.72$. Find the side and area of a regular decagon and then a pentagon. In example 28, the segments of a line divided into extreme and mean ratio were worked out.

$$\overset{X}{\overline{A \quad C \quad B}}$$

$$\frac{AB}{X} = \frac{X}{AB\text{-}X} \qquad X^2 + AB.X - AB^2 = 0$$



(Fig. 23)

A		2.93 = *AC*	
CI		1	
BI	1	7.65	
D	4.72	4.72	check

Let AB = one side of decagon.

$AB — 2.93 \qquad AO = 4.72$

$AC = \frac{2.93}{2} \quad 1.465$

$OC^2 — AO^2 — AC^2 = 4.72^2 — 1.465^2$
$= (4.72 — 1.465) \quad (4.72 + 1.465)$
$= 3.255 \times 6.185$

A	3.255	
B	1	6.185
C		
D		4.48 = *OC*

Area decagon $= \frac{1}{2} \quad 4.48 \times 2.93 \times 10 = 2.24 \times 29.03$

C	1	29.3 = area	Ans.
D	2.24	65.3	

$BD = AB$ and AD is a side of pentagon.

$OE = 2.93$
$EB = 4.72 — 2.93 — 1.79$
$EG = \frac{1}{2} EB = .895$

$GD^2 = 2.93^2 — .895^2 = (2.93 — .895) \quad (2.93 + .895)$
$= 2.035 \times 3.825 \qquad GD = \sqrt{2.035 \times 3.825}$

MAKING MATHEMATICS INTERESTING

By AUGUSTA BARNES
Girls' High School, Atlanta, Ga.

The question of interest is the foundation of all true methods of teaching. If the subject of mathematics could be made more interesting to the pupil, many of our most perplexing problems would be solved. How to secure a high mastery in the subject, how to influence pupils to continue the study, how to reduce failures, how to make mathematics learning permanent, how to develop skill in manipulation—these and many more problems would be easy of solution, if, indeed, they appeared as problems at all.

At present, when every teacher must defend his subject on the utilitarian basis, where knowledge is power, and the mind is a tool not an end in itself, where little thought is given to "truth for truth's sake," the mathematics teacher will have little difficulty in showing that his subject has a practical and useful value. He may be tempted to answer the question "What good do I get out of all of this?" as did Euclid centuries ago. "Give him a copper or two since he must make a profit out of what he learns." While he may at times have an inward revolt against the great emphasis which is being put upon the utilitarian side of all forms of education, still he will be able, with little difficulty, to defend his subject on the practical side. He can show the pupil that mathematics is useful in life, that it has a commercial value since every student will have money to spend, money to invest. He should know how to make his own calculations.

If, after showing him the practical value of the subject, the teacher can introduce him to some of the joys, some of the pleasures of mathematical learning, he shall have aroused a real human interest. The pupil no doubt has seen how a knowledge of history, language, or botany will make a substantial addition to his pleasure in life, how the joy he has in his reading is greatly enhanced because he understands historical allusions and can recognize quotations from classics. But little has been done in many of our schools to make him see the pleasure and the cultural value of mathematics. A step in this direction which has

been made by some schools is the establishing of Mathematics Clubs. In these clubs the pupil is encouraged to bring puzzles to be solved, items of interest from his reading, and plays written by the pupils and given at their meetings. Often the members will make accurate models of the solid geometric figures, others will develop geometric designs for needle work, and some will make graphs. Later, displays are made of this work, thus arousing interest and curiosity.

Quoting from "A Mathematical Club in a Girls' School" by I. M. Brown, in the Journal of Education (Brit.) we have the following: "We divided ourselves into three groups. One of these compiled a manuscript, or rather a typescript, book of mathematical puzzles and anecdotes. The second group searched the encyclopaedia and books of all sorts for stories of mathematics, and, finally, a volume of Lives of Great Mathematicians was completed. The third group wrote a history of arithmetic from earliest Babylonian times to the present day, paying special attention to the simplification of and quickening of methods and notations. Each of these books had many illustrations."

Mr. Brown writes further of his club: "We wrote a short play founded on 'Flatland.' There were six scenes and a prologue. The prologue performed the part of the chorus in a Greek play and introduced the characters. We made most charming programs, each with a diagram illustrating a scene in the play. . . . Judging by the frequent bursts of applause, the audience enjoyed itself quite as much as did the players. Even the youngest could recognize the geometrical propositions illustrated in the Flatland Children's games. We, with a purely mathematical program, had amused and entertained the school." This same club undertook to post a weekly mathematical story or puzzle in each upper classroom. These notices were generally taken from the books which they had compiled.

Another reason for the organizing of mathematics clubs is that it gives the brighter pupil an opportunity to do further work, and get an insight into the pleasures of mathematics. A club of this kind was organized by Frank C. Gegenheimer, Marion, Ohio. Membership was first confined to the junior mathematics classes, but soon after organization seniors who had taken junior mathematics were admitted to membership. The only qualification for membership was a desire to study mathematics. Meet-

ings were held once a week, at which at least one paper on a topic of mathematical interest was read by one of the members. Members of the class presented their proofs of the theorems and solution of problems which had been presented by the instructor or members of the class at the previous meeting. Lively discussions were frequently provoked. Members were encouraged to read articles in magazines relating to mathematics, and to give a report to the class. The trisection of an angle was taken up with interest, and the solutions which have appeared from time to time in magazines were examined.

The mathematics contest is another and very effective method of stimulating interest in the subject. The competitive spirit is a powerful stimulus to effort, since it develops enthusiasm and ability to a high degree on the part of the contestants. This has long been recognized as true in the athletic field, but only recently has it been tried out in any of the academic subjects. A contest of this kind was carried out very successfully between the Hyde Park High School and University of Chicago High School. A team consisted of the six best students from the first-year classes. The contest was both oral and written, the oral being open to the public. Tryouts for the teams were held; teachers also gave review contests between the various freshman sections. At the oral contest at which a large audience was present and remained throughout the entire contest, great interest and enthusiasm was manifested. Mr. Raleigh Shorling, in reporting on the contest, says: "It is simply a unique device which may, upon experimentation, prove to be of assistance in the solution of the problem of individual difference inasmuch as it will be a powerful stimulus to the enthusiasm and effort of the "A" students in mathematics."

The human mind has always found pleasure in puzzles, tricks and curiosities of all kinds. This trait is not common to any race or any period of history—it is innate in man. The problem of the fox, the goose, and the peck of corn and how to get them across the river was known in the time of Charlemagne, about 800 B. C. The hare and hound problem appears in an Italian arithmetic of 1460. Many of our present-day puzzles have come down to us from a much earlier time. The use of such puzzles and devices for arousing interest is not a new idea. Such things may be employed in our teaching as legitimately as much of the

material we are now using, and perhaps more effectively, to vitalize the subject of mathematics and show its connection with the ordinary affairs of life, to sharpen the faculties of the students, and to arouse in them an appreciation of the power and the beauty of mathematics.

Informal meetings of the mathematics class may be held where the time is largely devoted to mathematical games, tricks, puzzles. A part of the recitation period might be devoted to this phase of mathematics. The need of arousing and holding the students' interest is recognized by all wide-awake teachers, and whatever device will accomplish this will also advance the real work of the class room.

Some recreations for elementary algebra are included in the following: Algebraic fallacies (to prove $1 = 2, 1 = 0, 1 = -1$), history of the signs of operation, development of algebraic symbolism, games of chance, beginnings of algebra.

A few suggsted recreations in geometry are: chapters from the history of geometry, famous problems of geometry (squaring the circle, trisection of an angle, duplication of the cube); good luck symbols (the swastika, the monad, the cross, the Greek cross); the mathematics of common things (the watch as a compass, Sun Dials, the carpenter's square, telescope, compound mirrors); mathematical symmetry in nature and art; fallacies (to prove every triangle isosceles, to prove the circumference of a circle equal to its diameter). Many more suggestions will be found in the bibliography accompanying this discussion.

A subject which has been suggested for a meeting of a mathematics club and one that will arouse interest and bring the minds of the student to realize the universality of mathematics, its use and applications in life, is the subject "A Word Without Mathematics." The members should be encouraged to show what would happen in the business and industrial world, what changes would have to be made in our living and thinking if the whole knowledge of mathematics were erased from the minds of men.

An interesting experiment conducted by Mr. W. D. Reeve, of the University of Chicago High School and one that was successful in arousing interest and securing greater efficiency was an exhibit of high school mathematics. The plan briefly was as fol-

lows: Bulletin boards of work were placed in each of the mathematics classrooms. Upon these boards the work of the pupils was posted from day to day that other students as well as visitors might see what was being done.

The papers were chosen on the basis of neatness, importance of subject-matter, care in development of proofs, good independent work. These papers were referred to by instructors, and plans for improvement were developed by discussions. Any items from papers or magazines of importance in the world of mathematics were posted. A great increase in interest, a desire to improve the quality and the appearance of written work, as well as a gain in mathematical power compensated for the work in preparing the exhibit. A friendly spirit of rivalry added pleasure to the undertaking and increased the desire of pupils for independent research work. In addition to written work of pupils and items of interest found in reading, geometry models and special devices for use in the class room were made by pupils.

As to the organization of the material for the exhibit three plans were followed: First, material was arranged in certain definite groups, such as algebra and geometry; second, it was arranged in order of work done—first year mathematics, second year, etc.; third, a combination of both plans was used. The work was shown, not only by years, but also in special groups. In addition to these plans the exhibit was arrangd with reference to a certain pupil's work in mathematics and thus his improvement through the four years was shown. This exhibit was put in a permanent case and from year to year has been improved and changed. The added interest, inspiration, and efficiency gained by the pupils justified the experiment.

In introducing the subject of algebra or geometry to the pupil, or later in the course when interest seems on the wane, an informal talk from the teacher on the lives of the great mathematicians, or on the growth of the subject will inspire the pupil and vitalize the subject. The co-operation of members of the class in getting interesting historical facts and presenting them as a part of the recitation period, will tend to make the subject more alive to the pupil.

In the Girls' High School of Atlanta, Georgia, we have used successfully mathematical plays, the exhibit, and the contest

plan to increase interest. The following is a suggested plan for a contest. A class in geometry is divided into two sections with a captain for each side. One side has charge of the "interest program" for a week, then the other side in turn has it for a week. The members of each section are to bring in during their week as many applications of geometry and geometric designs as possible. These they may gather from their reading or make them themselves. They are posted each day on the bulletin board by a committee in charge of this work. Five or ten minutes of the recitation period is given daily to the section in charge, that they may report some interesting historical fact about the subject of geometry or about some famous mathematician. Section II has charge of this work the second week. At the end of the second week the entire class is asked to vote by ballot for the section that has been most successful in its undertaking. A total of these votes is kept by the teacher but not announced to the class till the end of the contest period which continues in this manner for six weeks. Thus the interest of the class is kept up till the end because of the uncertainty of the winning. At the end of the sixth week the last vote is taken and a total made of the votes for each side. The losing side entertains the winning side at a party.

The result of the contest will be to increase the interest in geometry, to make the subject more real and vital to the pupil, to show its applications to life.

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FOUR YEARS OF MATHEMATICS FOR COMMERCIAL PUPILS

By FREDERICK K. HUSSEY
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By four years of Mathematics I refer to grades 9, 10, 11 and 12. Just a word as to what previous preparation in mathematics pupils have when they reach the 9th grade. Their work in the 7th and 8th grades has been a General Course, bringing in and using simple Algebra in the solution of problems, also Geometric drawings and measurements. Business applications have been simplified in these grades and the time somewhat reduced. The above work has been outlined and supervised by Mr. H. C. Barber, Supervisor of Mathematics in Newton.

Investigation shows us that most boys go into an office as some kind of a clerk rather than into special lines as a Bookkeeper, etc. Business men want boys who are not speed fiends in addition, etc., but rather who are accurate; who can handle figures readily and intelligently; whose eyes and ears are wide open and who will take an interest in the business.

We cannot get accuracy by drill on arithmetic problems alone. Why? Because a boy gets disgusted with a monotonous continuation of what he has had for eight years in Grammar School. Also the boy in the Business Course can see no good in the x y z type of Algebra and we ourselves are at a loss to justify it. Instead of the hound chasing the hare which is three leaps in front of him, let the boys chase the dollars which are three leaps in front of them. One will say off-hand that there are not enough instances where business problems can enter into an Algebra Course, but I have found from my own experience during the past three or four years that there are plenty for us to use.

Copying figures, not letters, is more necessary now. Boys have a great deal of work in offices transferring figures from books.

Business men deal with figures every minute of the day. And as to the employees in the office (with the exception of a few specialized departments such as filing, etc., which are handled by women employees) it is through their handling of

figures that they earn their living. The boy who does his work intelligently and quickly is the one who attracts notice but he needs one other thing for his advancement to be assured, and that thing I shall speak of under the Mathematics for the 4th year.

The work of each year must be used so as to contribute something that is usable in the following year, rather than to be studied as separate years with a wall between.

The prospective young business man cannot picture himself (and rightly) working in an office talking xyz's and twirling triangles around his fingers. Both his Algebra and his Geometry must be made concrete instead of abstract. They must be used together with his business problems, dollars and s-e-n-s-e problems. Both Algebra and Geometry have an important and powerful place in Business Mathematics if used rightly. The donkey is a laughable beast when used to convey an aristocratic gentleman along Fifth Avenue, but this same donkey is a highly thought of animal when used to convey the same gentleman over the Andes.

I have made investigations in three ways to try and discover the kind of Mathematics *actually* used by business men in business offices. I have sent out a questionnaire to every male graduate of our school since its organization, twelve years ago, asking them to check from a list of mathematical operations those that they use themselves and in what connection they use them—giving their position. I am sending another questionnaire to representative business firms in and around Boston, asking them to check from an eight page group of sample problems such as are found in the standard Commercial Arithmetic books now on the market, the problems used in their office—and whether used for the arithmetical computations involved or for the General Business Information. The reason for this last I shall speak of again under the Mathematics for the Fourth Year.

Also I am personally visiting dozens of offices, both large and small, talking with the executives regarding what they want the boys to have for mathematical training in their particular office. So I think I have some foundation upon which to base my remarks.

Mathematics for the First Year

ALGEBRA

We must keep in mind that most pupils drop out of High School at the end of, or during, the tenth year. Investigators tell us that Algebra does not help them to stay in school. Therefore, our course must contain matter as valuable as possible to these pupils who leave, and I had this in mind when arranging the courses in Mathematics.

An intelligent working knowledge of the "Equation" and "Formula" constitute the backbone of the Algebra in this year's work. There is no necessity for very much factoring work, and no use of fractions involving more than one term in the denominator.

In connection with the Formula work of this year we should work in close conjunction with his Science work. We can help the boy understand his Science better if we use many of his Science Formulas concerning Levers, Pressures, Electric Currents, horse power of automobile engines, etc. These problems give us just about the right amount of fractional formula work that we want and give the boy a chance to see that he does use some of his algebra immediately. Further formulas come from his Geometry work in areas of rectangles, parallelograms, triangles, etc.

Profit and Loss problems and Interest problems are worked much more quickly and intelligently when solved by the Formula and Equation. I have given for several years a percentage problem to 9th grade boys about the middle of the year and asked them to use either arithmetic or algebra to solve it and about fifty per cent will use each method. Eighty per cent of those using arithmetic will get a wrong result while eighty per cent of those using algebra will get the correct result. They can grasp the entire subject of percentage very quickly through the medium of Algebra, because many percentage problems when solved by arithmtical processes seemed to be worked back-end-to, while by the algebraic processes he sees what he is doing and why he is doing it.

The average man solves his interest problems by $I = P \cdot R \cdot T \cdot$ and not by some special "Bankers Method." We must be careful not to give the boy the idea that he is studying Algebra so

as to have a substitute for Arithmetic, but to show him that he does continue to use his Arithmetic the same as before.

Graphs occupy an important part of this year's work. Not the abscissae and ordinate or the f (x) kind, but the graphs of business—the bar, rectangle, circle and line. The height of the bars or rectangles necessitates (rather than gives opportunity) for estimation and comparison and rounding-off of numbers. The same is true when studying the comparison of the monthly profits of two years by means of the line graph. When we come to the circle graph, probably the most used graph in business, this necessitates percentage computation and the use of the compass which brings us up to the need for further study of rectangles and parallelograms, extending the knowledge gained in the 7th and 8th grades. A man's expenses for a year are $10,000; $4,000 were spent for selling, $2,500 were spent for advertising, etc., etc. We must compute what per cent is each of the total and to do this we must study comparison of numbers. The value and use of a check is very plainly seen in problems like this—one column totals 100% and the other total is 360°. We do not stop with the mere finding of the dimensions of a tennis court whose length exceeds twice its width by 3 feet and having a perimeter of 210 feet. Who can conceive of anybody wishing to find the dimensions of a tennis court, or of any other rectangle, merely to satisfy his curiosity? What is the reason for finding the dimensions if not to do something with them? We go on and find the cost of taping the court at ¾¢ a foot; then we find the cost of putting a wire net around this court 10 feet back from the ends and 8 feet from the sides, with wire costing 4¢ a sq. ft. Then, also, we find the cost of resurfacing the court with clay at so much a sq. yd.

Since profits are based on selling price in practically every business now, computing these profits use as their basis the algebraic formula $C + G = SP$. A man buys goods costing $5,000 and wants to make a profit of 25% on the selling price, find the SP; ($\$5{,}000 + .25S = S$). Arithmetic tells us to let 75% = the cost then to find 1%, then 100%. But this method does not tell a boy why 75% = cost, while it is self evident by the formula.

Investment problems are worked on an algebraic basis. A man has $5,000 worth of Liberty Bonds which he wishes to

invest so as to receive an income of about $300. One investment will yield 6% and another 7%. How much must he invest each way?

A school gave an entertainment and raised $58.75, they charged 15¢ for children and 25¢ for parents. If they sold 325 seats in all, how many parents were present?

And so on with dozens and dozens of problems, constantly handling decimals and fractions.

When a boy solves a problem algabraically he has two things which are indispensable to his business training. The tearing apart of a problem and understanding the English language used (it is the sentence he does not understand and not the mathematics), and then the systematic arrangement of his work as he goes along. We shall see how this practice in systematic arrangement continues through the next two years' work.

The Business Course boy is interested in business problems or he would not be taking the Business Course. His interest is not held when he sees no connection between his Algebra and business problems, but he is interested with the kind of Algebra I have outlined because it seems to him sensible and usable. It is only human nature that none of us do as good a piece of work at anything we do not like, as we will if we do like it. Algebra *can* be humanized if we care to make it so. We study problems and use the algebra needed to solve them instead of teaching topics in Algebra. We simply use algebra to serve our purpose.

Mathematics for the Second Year

MAINLY PLANE GEOMETRY (No Text).

Geometry is, for us, essentially a study of drawings. Geometry gives us drawings illustrating our problems as we work them. It is the best aid to clear thinking and often helps to improve memory. The facts of geometry give us many useful things. The measurements in geometry give us practical computations, showing us the absurdity of 3.1416 as the value for Pi $= \pi$.

Painting a garage necessitates the handling of rectangles, and triangles; the hypotenuse rule is used to find the width of the roof which to be covered with roofing, and getting the cost of the painting and the roofing brings in fractions and decimals—the same as we noticed in the first year.

The estimation and comparison is brought up when we wish to find the cost of painting another garage or house. How much larger are the dimensions than those of our first garage? Compare the painting cost if the length, width and height are twice the first; will it cost twice or four times as much? The geometry back of this is the comparison of rectangles. We should actually work out several of these problems (since we have our eye on the computation work) and let the boys see how their answers compare and so come up to the theorem last rather than first. The same thing in the comparison of house lots. How large must boxes be for packing fancy apples averaging a certain size and containing a certain number of layers, etc,, and how much will the cost be per hundred. There is a vacant lot on the corner which men cut across on their way to the train in the morning—how much distance and time do they save—brings up the discussion of the Pythagorean Theorem as well as the boy finding how high in the air is his kite and *not* how long is his kite string; by the way, how does he find the point *directly* beneath the kite? Estimation, and approximate computation naturally is used with problems of this kind.

A boy likes to work an occasional "silly" problem and find the treasure which the robber hid equidistant from two trees. It is called "Locus" in geometry books.

When we study circles we always have the discussion of bicycle wheels and of oversized tires on automobiles. Here is a wonderful place to discuss gear ratio both in bicycles and in autos because boys can see some sense in comparing things of this sort rather than see how *much* larger 6 is than 5. By the way, I always make it a point to ask my First Year boys what is "Ratio" when we encounter our first Ratio problem (and we come across many of them in the first year) and I invariably get the answer that "Ratio is the relation between two numbers," but when I ask them what they mean by "Ratio" they do not know whether brother or sister, aunt or uncle. Isn't it funny how complicated we can make the simple things in Mathematics? Better no definition at all than one which does not make anything clear. To continue with circles, there is the plumber who puts a two-inch pipe in a man's house to take the place of two one-inch pipes. We ordinarily stop with finding how much too large the two-inch pipe was, but why not continue and find out

the thing the man would be most interested in, namely, how much more did it cost him that it should?

The comparison of numbers is constantly coming into our work without us dragging them in, so why not discuss them when they actually ought to be discussed instead of creating some fictitious occasion? The building of new running tracks in the athletic field and the number of laps for a mile in the indoor track, etc., give ample material for the computation with approximate value and including decimals and fractions.

It is the computation side of geometry that we are after for our Business Course rather than the proof side. The practical applications of Plane Geometry are comparatively few when compared with Solid Geometry, but if we stop thinking about getting the boy to prove that "the perpendicular from the right angle to the hypotenuse is the mean proportional between the segments into which the hypotenuse is divided and either leg is the mean proportional between the whole hypotenuse and that segment adjacent to the leg, etc.," then we can find far more than we thought at first.

Now, just a word as to the demonstration side of geometry. First a boy accepts the authority of other people. The second way of discovering facts is by experimenting. And now comes the only other way of finding truths, i. e., the field of logic. We do not discount the first two but now give a modest amount of demonstration. The study of logic tends to give a boy a logical argument and this is a most necessary requisite for a business man. The whole tendency now is to cut down the number of required theorems.

We teach enough proofs in geometry for every boy to see the necessity of a proof, and to give a continuation of a systematic arrangement of facts which he started in his algebra work. He automatically acquires the habit which he must use later when working in a business office. But, as I said a moment ago, the proof side is not paramount in a Business Course.

Mathematics for the Third Year

MAINLY SOLID GEOMETRY INVOLVING REVIEW OF PLANE AND ALGEBRA

(No Text)

Mathematics was formerly offered in the third year only to boys who intended to continue their business training at some

Business School or College. But the course for this year has worked out in such a way as to show its many advantages to all boys whether they continue their schooling or not, so now it will be given to all boys in the third year.

The first thing in this year's work is to teach the use of the Slide Rule. You may disagree with me as regards the actual use of the Slide Rule by business men—but my investigation has shown me that it is much used. It is coming to be used more and more every day, and such companies as "Keuffel and Esser" are putting on the market a special "Merchants Rule." Dozens and dozens of large offices in Boston are buying them to use in their office work. Each member of the class is provided with an inexpensive (dollar) but accurate rule and is taught to multiply and divide, both separate and continued, proportions, squares and square root. A boy can master these operations on the rule in a surprisingly short time, only a question of days. One strikingly valuable thing that the boy gains is that he must use his head instead of some "rule of thumb" method of placing the decimal point. If for no other reason than this I would recommend the use of the rule to Business Course boys. Many boys go to an office and are taught the Slide Rule—we, as educators, should show the business man rather than have him show us. This necessitates the use of "Significant Figures" and "Approximate Computation" which he has had already. He begins to see and understand the size of numbers in relation to other numbers, which is one of the two main uses of numbers by the business man. We use the slide rule in practically every computation throughout the entire year.

Now regarding the Solid Geometry part of the course. We take five solids: the Prism, Cylinder, Pyramid, Cone, and Sphere. We deal primarily with the Lateral Surface and Volume of these solids; the geometry proofs necessitating a review of plane geometry. In this way the proofs come very easily and readily to the boys. The analysis and systematic arrangement of data in the form of a proof is in this way continued, but again, as in the Plane Geometry work, it is not of primary consideration in a Business Course. It is useful to the extent that it habituates a clear and systematic arrangement of computation and an enormous amount of computation is done in this year's

work. Algebra straightens and lays out the computation. The Geometry gives us the logical part of the analysis.

The size of warehouse rooms and packing cases necessitates computation of and comparison of similar rectangles and prisms. Convenient size of boxes and cans necessitates computation of and comparison of prisims and cylinders. Manufacturing problems call for an unusually large amount of work with these five solids. But we are not stopping with this, but are continuing with the figuring of profits, interest on borrowed capital, discount to buyers, etc., which brings up problems of the wholesaler and then the retailer and so on. A manufacturer of hot water tanks is not only dealing with dollars and cents, but with volume and surface of cylinders as well. So when discussing a problem of this sort we go back to the beginning and tackle the Solid Geometry part of it and then continue with the profits on the sales, discounts to dealers, etc. The same is true of the man who manufactures tents, he is always with surfaces of prisms and pyramids as well as with profits and discounts. Digging of subways and tunnels does not consist merely of finding how many cubic yards of dirt is excavated, but they are usually walled with concrete or something that necessitated some expenditure as well as the digging. What would be convenient dimensions of a 15 gal. gas tank on an automobile opens a wonderful opportunity for estimation. Take the question of how much coal is saved when the steam pipes are wrapped gives us cylinders to start with, then transportation costs and dealer's costs. In every problem we are reviewing our plane and using our Solid Geometry. In short, we do not deal with frustrums, truncated prisims, etc., in every day life, but solids are always tied up in some way or other with the dollars and cents of business. So why not teach them as we actually use them in life?

Mathematics for the Fourth Year

Let us look at the contents of a Commercial Arithmetic book. In general they are pretty much the same in all books. I was interested, a short time ago, to compare a present day book with one printed in 1894 and I was much surprised to find the table of contents essentially the same. The first half of these books is given to a review of Addition, Subtraction, Multiplica-

tion and Division, Decimals and Fractions. Then comes Profits and Losses, Discounts, and Interest of all varieties—Simple, Annual, Compound, Exact, Bankers Method, and what not. The latter part of the books is usually given to the treatment of such subjects as Insurance, Taxes, Annuities, Negotiable Papers, Depreciation, Partnership and Corporations, Stock and Bonds, Equation of accounts, Partial Payments, Ratio, and Proportion, with usually an entire chapter, separate by itself, dealing with Short Cuts. Surely any boy who can master the contents of a book such as the above, could step from High School directly into a Business Manager's position.

Now the first part of these books are certainly justified as Arithmetic, and is taught for its arithmetical content. But how about the latter part, can those topics be justified as Arithmetic and taught from an Arithmetic text book? Are they taught for their arithmetical content or for the "General Business Information" they convey? Is the Arithmetic of primary or secondary importance? Business men are unanimous in saying that these subjects are important for the general information they carry. After all there isn't much *mathematical* thinking called for in computing a tax or an insurance premium. As a matter of fact the computation in many of these topics, as in Taxes, Insurance, Compound Interest, etc., is done by reading it from a table. The arithmetic part of Bank and Sight Drafts is never a puzzle to boys, but ask them whether a Bank Draft is used to pay or to collect a bill and fifty per cent of them will answer each way, although they have dealt with Drafts for two or three years in their bookkeeping work.

Also business men, in the majority, are emphatic that boys should be taught these topics which convey a General Business Information.

As one of the officers of a large paper concern in Boston told me, after discussing the question with the head of their Statistical Department, "A proper grounding in the ability to figure and the acquisition of a general theoretical knowledge of business problems in addition to the mere foundation of Mathematics" is what they want in a new-comer to their offices. The eight years of grade school work and the previous three years of such work as I use and have just outlined leave little doubt of a boy's ability to figure. Now then, how to teach these sub-

jects of General Business Information. To be sure he gets some general information through his bookkeeping work, but there the emphasis is naturally placed and the boy's mind concentrated upon the *"How"* and *"What To Do"* rather than upon the reason *"Why"* a Bank Draft is being handled in his problem.

We take problems concerning the Manufacturer, the Wholesaler, and the Retailer—and it is hard to discuss any one of these without one or both of the others coming in—and actually discuss the different things relative to the business—as the ways of paying a bill and of collecting a bill--the discounts offered and why, the estimating and figuring of profits, depreciation, bad debts, fire insurance, sinking funds, etc., and while discussing these things which involve arithmetical computation we actually do the figure work just as the boy will be called upon to do it in an office, using all short cuts, that are actually used, when they should be used and not teach them separately. In this way we teach Statistics, not as a topic, but throughout the course—Alignment Charts, Wage Charts, Logarithmic Graphs, showing the comparison of *amount of change* with *rate of change*. A more advanced use of the slide rule is taught, as used in such problems as this: The price of an article is $12.65, a series discount of 40-50-10% is given and a profit of 25% is desired—to find the selling price. In this manner we can teach those topics which are not fundamentally arithmetic, and have them understood if we do not drag them in as an excuse to find something upon which to practice our knowledge of the arithmetic we have learned. The computation work comes naturally when discussing the business problems.

Three years of work in the High School Course grounding the ability to figure and the last year perfecting and polishing this ability in a practical manner and sending out the boys with some knowledge of business transactions which gives them confidence in their work and so that they may have a chance to keep their eyes and ears open to notice what is going on in the business about them. They are equipped to hold their jobs because they know what they are doing and also are better equipped to advance.

MATHEMATICAL GAMES

By ALFREDA RASTER
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Most teachers of high school algebra at some time in their careers have to answer the eternal question, "What shall I do today?" And that question usually comes when the teacher is busy with about a million and one other duties. Frequently the teacher is asked to give suggestions to the school mathematics club for a short notice program. At such times the following games may be used to advantage. The rules may be varied at the teacher's pleasure to serve special purposes, or so that they will be better adapted to the teacher's personality. Suggestions for varying these rules to suit different occasions might easily fill a large volume, since the people involved really make the game, adding unwritten rules by the score. The bare rules answer the "what" of the games, rather than the "why, where, when, and how."

Old Maid in Algeba. This game is exactly the same as that of Old Maid, except that the pack of cards is different. Several types of packs of cards may be made. Special occasion packs may add interest in the game. The teacher or the president of the mathematics club may use his individual preference in making these cards.

The pack should consist of nineteen cards—one "old" one, and the rest of which may be "matched." An odd card is one which is not united to any other by a mathematical principal. Two cards which match are cards which are united by a mathematical principle. For example, if the basis for matching is that the cards which match must have a common root for the equations which appear on them, then $2x = 6$ matches $3x = 9$, since both of these equations haxe $x = 3$ as a root. However, the equation $2x = 6$ does not match $3x = 12$, since the roots of these equations are different. Other bases for matching are fractions having a common value when simplified; indicated products, sums, quotients and differences, which have the same result when the indicated operations are performed; and names of expressions to be matched with an expression which represents only part of the original class.

(*Directions for Playing Old Maid in Algebra*). To play the game, deal the cards face down, one at a time, until all are given out. Each player picks up his cards. If he can match any of the cards in his hand, he lays them aside for the remainder of the game.

When this has been done, the dealer, without exposing his cards, extends them fan-like to the player on his right, who draws one card. If this player can then match any two of his cards, he does so and lays them aside. Whether or not he succeeds in matching, he extends his cards to the right, and that player tries to match some of his cards.

Each player pursues the same course, drawing and matching and extending, until no further matches can be made. There will now be only one card left. The player who holds this card is the Old Maid.

Blackboard Relay. The class is seated with an even number of players in each row. Those in front, at a given signal, run forward and write on the board at the front of the room the first word of a given rule. Upon finishing the word, each player returns at once to his seat, handing his chalk as he does so, to the second player in his row. This player at once runs to the board and writes the second word after the first one. In this way the rule is completed, the last player in each row being required to add the punctuation marks.

Variations. Variations of Blackboard Relay are frequently more successful than the game itself. Two are given below.

1. Before the class commences, the teacher may write as many problems on the board as there are pupils, placing as many problems in a column as their are pupils in a row. Each of the pupils in the front row solves the first problem in his column, and each person in the second row from the front solves the second problem in his column.

2. In a mathematics club, each pupil may be required to write three words on the board about his mathematics class. Each player must write words that will "make sense" with that which was previously written, but he is not required to finish the sentence unless he is the last one in his team to run.

Crossed Words. The teacher or one of the pupils makes a list of words that are more difficult than the average, or which are

of considerable importance in algebra. One of the pupils changes the letters about, and makes sufficient copies to supply each pupil of the class with one. The class is invited to write after each word the word from which it was derived. A few examples are:

ootr	root	emirp	Prime
noitequa	equation	aidrlac	Radical
opewr	power	rpgha	graph
rtcfoa	factor	eemrsb	members
tarnalio	rational	dttniyei	identity

In a given time the person guessing most of the words correctly wins the game.

Zip-Zip. One of the pupils goes to the board, and as he stands with his back to it, the teacher writes a rule or formula on the board above his head. The word should be visible to the class, but not to the pupil.

The pupil at the board asks questions of various people which must always be answered by "Yes" or "No." If no one is addressed, the entire class may answer.

As soon as the pupil at the board guesses what has been written, he chooses another pupil to take his place.

Simon Says. One person is chosen as leader. The leader says, "$2b—5$ is an equation," whereupon he stands erect. The players do likewise. Then the leader says, "$2b — 10$ is a binomial/", whereupon he stoops, the others seating themselves.

If at any time the leader says something true and stands, the players must be seated, and if he should say something false and stoop, the players should not imitate his movement.

Any player imitating him under these circumstances must take the leader's place.

"Yes-and-no" questions may be used in a similar way.

Problems having $6x$ for their results may be the signal to be seated; and those not having $6x$ for their results would then be the signal for rising.

The surprise and fascination of this game result from the quick thinking of the leader.

This game is also called "Stoop," "Teacher," and "Tattler."

Buzz. "The product of the sum and difference of two numbers is the difference of the squares of the numbers." This is a rule which may be used.

One of the players starts the game with the first word, "The." The next player says, "Product," and so on until the word "Number" is reached. Instead of saying "Number," however, the player says "Buzz." Whenever a player says "Buzz," when he should have said one of the words of the formula, or when he says "Number," instead of "Buzz," he is dropped from the game, and the formula is begun over. When the player is dropped from the game, he retains his seat, but is silent. Should a player skip his turn because he expects one who is dropped to answer, he also is dropped. The last person to be dropped wins the game.

The Game of Math Shark. The pack of cards contains twelve sets of matching cards with four cards in each set, making forty-eight in all. The cards match on principles which were explained in "Old Maid in Algebra."

All the cards are perfectly shuffled before the game. They are now divided equally among the group. The aim of each player is to get as many matching cards as he can.

When he receives his share of cards, the player examines them and if he finds an entire book (four cards), he lays them aside. When no one can form further books from the cards in hand, the drawing begins. Each person draws from the left-hand neighbor. If by this drawing he should form a book, it is laid aside. When all of the cards in the pack have been formed into books, the game is at an end. Then follows the counting of the books. Each book correctly formed counts one point. An incorrect book causes the player to lose five points.

If several packs are used on different days, they may be played for championship in a tournament. The champion is the *math shark*.

THE RELIABILITY OF TEACHERS' MARKS

By WALTER O. SHRINER[1]

The Aim—It was for the purpose of ascertaining the reliability of teachers' marks, under normal class-room conditions in the grading of the ordinary subjective type of written examinations, that this study was made.

Previous Studies.—During the past fifteen years a number of scientific studies have been made which dealt with different phases of this problem. However, the present study is in no way aimed at a duplication of any one of these former experiments, but attacks the problem from a different angle and results in somewhat different conclusions.

For the purpose of showing what has been done and to point out wherein certain conditions, which are felt to be vital, were not considered in the former studies, it is deemed advisable to briefly review a few of the best known of these former experiments.

1. Professor Jacoby of Columbia University reported an investigation in 1910 under the title, "Notes on the Marking System in the Astronomical Course at Columbia College,"[2] which is similar to the present one except that it was conducted on a smaller scale and in the collegiate field.

Professor Jacoby submitted for independent grading eleven astronomy papers consecutively to six professors in his department. The results as he presented them are given in Table I of this report. The first column of Table I gives the individual examination papers in order. The remaining columns give the grades ascribed to the various papers by the respective professors, designated by the letters A, B, C, D, E and F.

For the purpose of determining the coefficients of correlation for the sets of marks given by the respective professors according to Spearman's Rank-Difference Formula, the writer prepared Table II which gives the corresponding ranks. Table II is to be read in a manner similar to that of Table I.

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[1] Now a graduate student at the University of Michigan.

[2] See Science Magazine, Vol. 1911.

Table III was also prepared by the writer, and gives the correlation coefficients for the ranking determined by the grades of each professor with that of each of his colleagues. The first column of Table III gives the professors as designated by the letters A, B, C, D, E and F. The remaining columns give the correlation coefficients of the professor, as indicated at the head of the respective columns with the corresponding professors in the first column.

TABLE I

SUMMARY OF THE GRADES OF SIX PROFESSORS (PROF. JACOBY'S STUDY) GIVEN ELEVEN ASTRONOMY PAPERS

Paper	TEACHER					
	A	B	C	D	E	F
No. 1	9	9.0	8.5	7.2	9.0	7.3
No. 2	7	6.6	7.0	5.9	6.0	6.5
No. 3	9	9.0	8.8	7.2	8.0	8.0
No. 4	10	9.4	9.9	8.0	10.0	9.2
No. 5	7	6.2	6.7	5.8	7.0	5.9
No. 6	10	9.8	9.6	7.6	10.0	9.5
No. 7	6	5.8	6.3	4.6	7.0	5.4
No. 8	9	9.3	9.7	8.0	9.0	8.8
No. 9	8	5.7	9.0	6.7	10.0	8.7
No. 10	10	8.5	9.1	6.2	9.0	9.0
No. 11	9	9.0	9.5	6.1	8.0	9.0

TABLE II

SUMMARY OF THE RANKS AS DETERMINED BY THE GRADES GIVEN THE ELEVEN ASTRONOMY PAPERS ABOVE

Paper	TEACHER					
	A	B	C	D	E	F
No. 1	5.5	5	8	4.5	5	8
No. 2	9.5	8	9	9	11	9
No. 3	5.5	5	7	4.5	7.5	7
No. 4	2	2	1	1.5	2	2
No. 5	9.5	9	10	10	9.5	10
No. 6	2	1	3	3	2	1
No. 7	11	10	11	11	9.5	11
No. 8	5.5	3	2	1.5	5	5
No. 9	8	11	6	6	2	6
No. 10	2	7	5	7	5	3.5
No. 11	5.5	5	4	8	7.5	3.5

TABLE III

CORRELATION COEFFICIENTS FOR THE RANKING AS DETERMINED BY THE SPEARMAN FORMULA

$$r = 1 - \frac{.6SD^2}{n(n^2-1)}$$

(where D=difference in rank and n=no. of papers.)

Teacher	TEACHER A	B	C	D	E	F
A	---	.802	.825	.750	.737	.907
B	.802	---	.764	.800	.491	.730
C	.825	.764	---	.821	.750	.925
D	.750	.800	.821	---	.775	.693
E	.737	.491	.750	.775	---	.768
F	.907	.730	.925	.693	.768	---

It is interesting to note that all of the correlations are positive; that with the exception of two, all are above .7. It should also be remembered that in the original investigation no attempt was made to rank the papers and to compute the correlation coefficients, and as a result, many of the original papers were given the same grade by the same professor. This naturally affects the coefficients as given in Table III, since all papers given the same mark by the same professor were given the interpolated rank of the group. It is quite evident, since all the coefficients are decidedly positive, that if an exact ranking had been required a finer and a trifle higher correlation would have resulted.

The conclusions which Prof. Jacoby drew from the results given in Table I are as follows:

(1) There existed a very close accord in the marks given by the various professors.

(2) It would appear . . . . that in the case of an exact science in the collegiate field, the marking system was more precise than some critics maintained.

Among the limitations of this study in the drawing of general conclusions to be applied to the reliability of marking within the public schools and in the secondary schools in particular were felt to be the following:

(1) The investigation was conducted by teachers with papers on subject matter outside the public schools.

(2) The number of papers used in the investigation was limited to eleven which is a smaller number by far than is found in the normal class-room situation in the public schools.

(3) The number of professors grading the papers was limited to six, all of whom were experienced, of the same department and most probably employing the same standards.

2. Probably the best known and most cited study of this nature which has been published is that conducted by Starch and Elliott in 1911-12. Two distinct experiments[1] were made by them.

In one of these, Starch and Elliott procured two English papers written by two students on a final examination and had facsimile reproductions made and sent out to each of the high schools included in the North Central Association of Colleges and Secondary Schools, with the request that the papers be marked by the principal teacher of English on the scale of one hundred per cent. The results of 195 gradings, which were returned, showed a wide variation in marks (from 35 to 40 points) for the same paper.

In the other experiment, the same procedure was followed except only one geometry paper was used. The results of grading, based on the 116 tabulations returned, showed a similar wide variation in marks. The range was from 30 to 90 points.

Starch and Elliott held that in the face of such facts only one conclusion is possible, *i.e.*, that under normal conditions the marks assigned to examination papers by teachers are very unreliable.

Among the limitations of this combined study in the drawing of general conclusions to be applied to the reliability of teachers' marks were felt to be the following:

(1) Only one paper was used for the investigation in the case of geometry and two papers in the case of English. Under normal class-room conditions, there are usually more papers graded at a time and it is quite probable that this factor plays a large part in reliability of the marks given.

(2) Only the leading or principal teacher of the special subject in a given high school was asked to submit a grade. In order to draw general conclusions the work of teachers of various lengths of service, and degrees of skill should be considered.

3. Not long ago, Dr. Ben D. Wood of the Institute of Educational Research, Columbia Teachers' College, made a study of the reliability of markings on College Entrance Board examination

[1] Starch & Elliott—School Review, Vol. 21, pp. 254-59.

papers in algebra and geometry. In this investigation 413 algebra papers and 396 geometry papers were each scored twice by different readers. All of the readers had received the same general instructions as to the credits and standards to be applied, and had been working together several days under the eyes of vigilant chiefs who were anxious to secure uniformity in the marking. However, the grading and regrading of the papers involved in this study were made in total independence of each other.

The results showed that the correlation coefficients as determined by the Pearson Formula were very high. For algebra this reliability coefficient was $r=.986$ ($n=413$); for geometry it was $r=.956$ ($n=396$).

The conclusions drawn by Mr. Wood were to the effect that under the conditions above given, the reliability of the first markings was so high as to make rereading a waste of time, energy and money.

Among the limitations of this study in the drawing of general conclusions regarding the reliability of teachers' marks in the public schools, were felt to be the following:

(1) The readers were especially trained for the work in this study and, therefore, should not represent normal class-room conditions.

(2) The conditions were quite controlled as to the weighing of the different elements, the basis for scoring, etc. These, again, are hardly representative of ordinary conditions.

Present Study Program.—It was deemed advisable in the conduct of this study to carry the investigation into three high school departments under as normal conditions as could be made possible. Twenty-five semester-final examination papers were procured from each of three classes in Freshman English, Sophomore Algebra and Sophomore Geometry respectively. These classes were composed of unselected pupils in the Ann Arbor High School where the pupils are not classified into sections on the basis of intelligence scores or achievement records.

The original papers in all cases were sent out to teachers of varied lengths of experience and degrees of skill in those specific subjects. Absolutely no instructions were given as to how the different elements were to be weighed or scored. In the reading of the papers all teachers were cautioned to make no corrections on the original manuscripts so as to avoid influencing the marks

of a subsequent reader. The importance of grading the papers and of filling out the especially prepared report blanks independently of any other teacher was stressed.

In submitting their reports, all teachers were requested not only to give the grades in terms of per cent on the scale of 100, but also to rank the papers consecutively in order of merit. The latter was desired in order to facilitate the computation of the correlation coefficients by the Spearman Rank-Difference Formula.

It is regretted very much that in the course of the study that the geometry set of papers has been lost in transportation. It is hoped that a new set of geometry papers can be obtained and the experiment eventually completed as originally planned.

To date, the grade reports of ten different teachers have been received from various parts of the country on each of the algebra and English sets. While this experiment is still in progress, the results thus far obtained present some very interesting evidence.

All the computations for finding the coefficients of correlation in the marks of the various teachers submitting grade reports were made by the writer using Spearman's Rank Difference Formula,

$$r = 1 - \frac{6SD^2}{n\,(n^2 - 1)}$$

(where D = the difference in rank, n = the number of items ranked.) The computations were checked for accuracy and it is felt that the numerical results in this connection are quite reliable

The Report Proper.—In this preliminary report the results of the investigation thus far obtained will be presented in two sections according to the specific subjects involved, and each followed by a general summary.

Algebra

The algebra examination, from which the papers used in this study were taken, was one hour in length and consisted of nine problems of different types and degrees of difficulty. All ex-

amination papers were collected by the teacher at the end of the hour, regardless of whether or not the pupils had finished writing. The list of questions used is being submitted herewith.

The results of the different readings made by the ten algebra teachers were compiled from the report sheets and presented in terms of the per cent marks and the ranks in Tables V and IV respectively.

The first column of Table V gives the different papers which are designated by letters of the alphabet. The next ten columns give the per cent grades assigned to the corresponding papers by the various teachers designated in the headings by the ten Roman numerals. The last three columns give the range in per cent grades for the different papers in terms of the highest, lowest and the difference between the two respectively. Table IV is read in a manner similar to that of Table V and gives the results of the ranking.

Table VI presents the correlation coefficients between the rankings of the papers of each teacher with that of every other teacher. The first column of Table VI gives the teachers as designated by the Roman numerals, while the other columns give the correlation coefficients of the teacher indicated in the headings with the corresponding teachers in the first column.

Table VII was prepared for the purpose of showing the per cent grade distributions made by the respective teachers. The first column gives the teachers as designated by the Roman numerals. The remaining columns give the number of papers given the per cent grades as indicated in the headings by the corresponding teachers in the first column. For example, Teacher I gave 2 papers grades between 50 and 59, 2 papers grades between 60 and 69, 7 papers grades between 70 and 79, etc. At the bottom of the table the medians are given.

Some of the outstanding facts as revealed by these tables on the marking situation are as follows:

1. **The correlation coefficients as determined from the different rankings are very high.**

   All of the correlation coefficients as given in Table VI are above .6; 39 are above .7; 27 are above .8, and 11 are above .9. None of the correlations went as low as did those in the early study of Prof. Jacoby. This would seem to indicate that the marks of a teacher given to the papers of an average-sized class under normal conditions is very reliable.

TABLE IV

SUMMARY OF RANKING OF TWENTY-FIVE ALGEBRA PAPERS BY TEN TEACHERS OF ALGEBRA

Paper	TEACHER										RANGE		
	I	II	III	IV	V	VI	VII	VIII	IX	X	High	Low	Dif
A	7	8	7	2	6	6	2	2	1	2	1	8	7
B	16	21	23	15	23	23	12	21	22	24	12	24	12
C	4	6	3	5	5	3	3	4	6	5	3	6	3
D	21	20	21	25	14	21	18	13	13	15	13	25	12
E	12	9	14	16	13	13	7	14	14	13	7	16	9
F	2	5	5	4	4	4	1	3	5	3	1	5	4
G	23	22	22	22	19	12	16	23	24	20	12	24	12
H	6	4	4	10	8	7	2	7	7	7	2	10	8
I	8	10	13	8	7	9	15	11	10	11	7	15	8
J	9	7	9	7	9	8	5	9	11	10	5	11	6
K	20	18	17	17	12	18	17	15	12	12	12	20	8
L	11	11	12	19	10	10	8	10	9	9	8	19	11
M	18	15	19	9	22	16	13	21	21	21	9	22	13
N	1	2	1	1	2	1	2	1	2	1	1	2	1
O	24	24	24	23	21	17	19	24	23	23	17	24	7
P	19	17	15	11	15	14	11	18	20	19	11	20	9
Q	10	16	10	14	18	19	14	12	15	16	10	19	9
R	17	19	18	12	16	20	9	16	17	17	9	20	11
S	14	14	8	20	17	15	7	8	8	8	7	20	13
T	15	13	16	21	11	11	6	18	19	22	6	22	16
U	5	3	6	3	3	5	4	5	3	4	3	6	3
V	22	23	20	18	20	24	10	20	18	18	10	24	14
W	25	25	25	24	25	25	20	25	25	25	20	25	5
X	13	12	11	13	24	22	13	17	16	14	11	24	13
Y	3	1	2	6	1	2	2	6	4	6	1	6	5

TABLE V

SUMMARY OF THE GRADES GIVEN TWENTY-FIVE ALGEBRA PAPERS BY TEN TEACHERS OF ALGEBRA

Paper	TEACHER										RANGE		
	I	II	III	IV	V	VI	VII	VIII	IX	X	High	Low	Dif
A	96	95	89	100—	94+	90·	96	100	100	100	100	89	11
B	77	69	59	62	44+	45+	70	59	60	57	77	44	33
C	98—	97	94	94	94+	93+	94	98	96	99	99	93	6
D	70	72	66	34	62—	55	62	75	73	71	75	34	41
E	82	89	78	60	67—	67	84	74	72	71	89	60	29
F	98+	98—	92	95	94+	93	98	99	98	99	99	92	7
G	62	68	65	55	56—	67	65	57	60	63	68	55	13
H	97	98	93	72	83+	88	96	90	94	93	98	72	26
I	88	88	79	75	89—	83	66	76	78	86	89	66	23
J	87	96	84	80	83+	85	89	86	77	77	96	77	19
K	72	78	74	60	78—	60	64	71	74	72	78	60	18
L	83	88—	80	58	78+	80	78	81	81	83	88	58	30
M	75	81	71	75	44—	65	68	64	65	62	81	44	37
N	98+	99	96	100	95—	95	96	100	100	100	100	95	5
O	55	64	56	45	50+	60	50	54	54	55	60	45	15
P	74	79	77	70	61+	67	72	67	65	63	79	61	18
Q	84	80	83	65	56—	60	67	75	72	70	84	56	28
R	75	77	72	67	61+	55	77	70	68	67	77	55	22
S	80	82	87	56	56—	65	84	88	90	86	90	56	34
T	77	85	76	56	78—	70	86	67	65	60	86	56	30
U	98	99—	91	98	95—	93	92	96	99	99	99	91	8
V	62	68—	70	58	50+	45	76	66	67	66	76	45	31
W	55	54	50	35	22+	40	48	43	45	44	55	22	33
X	80	87	81	65	39—	55	68	69	72	71	87	39	48
Y	98	100—	95	90	100	95	96	96	98	97	100	90	10

TABLE VI

SUMMARY OF THE RANK CORRELATION COEFFICIENTS OF TEN TEACHERS—ALGEBRA PAPERS

Teacher	I	II	III	IV	V	VI	VII	VIII	IX	X
I	---	.950	.936	.829	.825	.825	.708	.906	.864	.829
II	.950	---	.932	.816	.855	.874	.734	.912	.859	.855
III	.936	.932	---	.770	.801	.858	.729	.924	.893	.904
IV	.829	.816	.770	---	.687	.713	.632	.728	.703	.719
V	.825	.855	.801	.687	---	.908	.656	.873	.872	.847
VI	.825	.874	.858	.713	.908	---	.676	.796	.766	.786
VII	.708	.734	.729	.632	.656	.676	---	.704	.664	.661
VIII	.906	.912	.924	.728	.873	.796	.704	---	.978	.968
IX	.864	.859	.893	.703	.872	.766	.664	.978	---	.978
X	.829	.855	.904	.719	.847	.786	.661	.968	.978	---

TABLE VII

SUMMARY OF THE DISTRIBUTION OF GRADES GIVEN TWENTY-FIVE ALGEBRA PAPERS BY TEN TEACHERS

Teacher	Frequency according to grades								
	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99	100
I	0	0	0	2	2	7	7	7	0
II	0	0	0	1	4	4	8	7	1
III	0	0	0	3	2	8	6	6	0
IV	0	2	1	5	6	4	1	4	2
V	1	1	2	5	4	3	3	5	1
VI	0	0	3	3	8	1	4	6	0
VII	0	0	1	1	7	5	4	7	0
VIII	0	0	1	3	5	6	3	5	2
IX	0	0	1	1	7	7	1	6	2
X	0	0	1	2	6	7	2	5	2
Median	0	0	1	3	6	5	4	6	1

TABLE VIII

SUMMARY OF THE DISTRIBUTION OF GRADES GIVEN TWENTY-FIVE ENGLISH PAPERS BY TEN TEACHERS

Teacher	Frequency according to grades								
	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99	100
I	0	0	0	4	4	6	9	2	0
II	0	0	0	1	2	10	10	2	0
III	0	0	0	1	1	9	10	4	0
IV	0	0	0	1	4	7	8	5	0
V	0	0	0	3	3	6	11	2	0
VI	0	0	0	1	5	8	9	2	0
VII	0	0	0	2	6	10	4	3	0
VII I	0	0	0	2	6	6	8	3	0
IX	0	0	0	3	4	9	6	3	0
X	0	0	0	3	7	3	7	5	0
Median	0	0	0	2	5	7	8	3	0

2. There exists a hierarchical order among the correlation coefficients for the different teachers.

   In Table VI, for example, in the column and row corresponding to Teacher VII, all correlations of this teacher with the other teachers are below 0.8. This naturally leads to the conclusion that Teacher VII is weak in the ability to properly evaluate papers in algebra. It also suggests the need of giving this phase of school work more consideration in the teacher training institutions.

3. The range in values assigned to different papers within the set varies greatly.

   In Table V, for example there was a variation in range for paper N of only 5 points, while in the case of paper X it was 48 points. In general, the range seemed to be less for the exceptionally good papers, but on the other hand, it does not follow that the greatest range occurs with the poorer papers. Paper O, one of the poorest papers, as shown by the marks of the ten teachers, had a range of only 15 points, while Paper X, which on the whole was accorded a much better rank, had a range of 48 points.

4. The distribution of grades, on the whole, did not approximate the normal distribution curve.

   In Table VII it is shown that in almost every case the greatest number of pupils received marks between 90 and 100. It should be remembered that the last column does not represent a full interval of ten points and for the purpose of interpretation could well be combined with the 90-99 column.

   This is probably the fault in part of the examination itself in that it was of such a nature that the members of the class could either pass all of it or little of it.

While this study was not directed toward the determination of the value of the examination itself, but the reliability of teachers' marks, it is evident that this other phase deserves careful consideration on the part of class-room teachers.

Table VII also suggests the importance of calling the attention of teachers to the matter of distribution. It would seem in harmony with good administrative practice to have teachers report monthly grades, etc., in some such form as given in Table VII for the purpose of calling their attention to their distribution tendencies.

English

The English examination, from which the papers used in this study were taken, was of the comprehensive type and consisted of elements in various degrees of difficulty. The list of questions and directions used is being submitted herewith. It should be remembered that no instructions concerning the weighing of the various parts in grading were given.

The results of the different readings made by the ten teachers of English were compiled from the data given in the report sheets and are presented in Tables VIII, IX, X, XI and in precisely the same manner as were the results on algebra in Tables IV, V, VI, and VII respectively.

Some of the outstanding facts, as revealed by the tables on results in English, are as follows:

1. The correlation coefficients as determined from the different rankings are exceedingly high.
   All of the correlation coefficients as given in Table XI are above .7 and all except 4 are above .8. There are 8 correlations above .9. This would seem to indicate strongly that the marks of a teacher given to the English papers of an average-sized class under normal conditions are very reliable.
2. There exists a hierarchical order among the correlation coefficients for the different teachers.
   In Table XI, for example, in the column and row corresponding to Teacher VII three of the coefficients are below .8 and none are above .9. This naturally leads to the conclusion that the reliability of Teacher VII's marks is less than that of the other teachers. It might be well to state that Teacher VII is quite inexperienced, although also are two of the others whose correlations were high.
3. The range in values assigned to different papers within the set varies greatly.
   In Table X, for example, there was a variation in range for Paper Q of only 4 points, while in the case of Paper S it was 29 points. This is sufficient to show that to draw conclusions concerning the reliability of teachers' marks it is not safe to limit the investigation to a single paper. In general the range seemed to be less for the exceptionally good papers, but on the other hand, it does not follow that the greatest range occurs with the poorest papers. Paper S shows the greatest range with 29 points and yet the median rank given it by all ten teachers is 16.
4. The distribution of grades, on the whole, approximates the normal distribution.
   In Table VIII, it is shown that the mode as a rule varied between the 70-79 and the 80-89 groups. No grades were given below 50 and no papers were marked perfect.
5. There exists a greater range among the algebra coefficients of correlation than for those in English.
   For example, there were six below .7 in algebra and none in English. On the other hand, there were eleven above .9 in algebra and only seven in English.
   There are several possible explanations for this situation although no one of them is offered as being absolutely valid.
   (1) It is quite probable that the English examination provided for more elements to be considered in the reading and, therefore, any differing in the evaluation of a single element would not affect the correlation to so great an extent.
   (2) There may have been teachers with less ability for evaluating papers in the algebra groups than in the other.
   (3) It may be more difficult to properly evaluate papers in algebra than in the case of English because of the difference in the natures of subject matter.

TABLE IX

SUMMARY OF THE RANKING GIVEN TWENTY-FIVE ENGLISH PAPERS BY TEN TEACHERS OF ENGLISH

Paper	TEACHERS										RANGE		
	I	II	III	IV	V	VI	VII	VIII	IX	X	High	Low	Diff
A	22	20	18	15	17	15	13	20	24	21	13	24	11
B	13	13	10	11	15	18	18	15	15	16	10	18	8
C	23	24	21	23	22	22	19	23	20	23	19	24	5
D	20	22	16	18	20	20	17	19	14	18	14	22	8
E	10	8	9	7	5	9	11	8	10	10	5	11	6
F	9	7	12	12	8	11	3	11	9	9	3	12	9
G	8	5	5	3	9	6	1	5	6	6	1	9	8
H	17	19	17	14	18	16	10	16	19	15	10	19	9
I	25	25	25	24	25	25	24	18	23	20	18	25	7
J	11	10	4	9	11	12	15	4	4	11	4	15	11
K	3	4	3	6	2	4	5	9	3	5	2	9	7
L	18	18	19	20	13	17	14	24	17	17	13	24	11
M	14	16	15	19	19	14	20	12	11	13	11	20	9
N	19	21	22	22	21	23	21	17	21	22	17	23	6
O	15	17	13	17	16	13	16	22	18	14	13	22	9
P	24	14	24	25	24	21	25	25	25	25	14	25	11
Q	2	2	2	1	1	1	2	1	2	2	1	2	1
R	5	11	6	13	12	7	8	7	8	3	3	13	10
S	7	15	20	16	14	19	22	13	16	24	7	24	17
T	16	12	14	10	4	8	9	14	12	8	4	16	12
U	12	9	8	8	6	10	12	10	13	12	6	13	7
V	21	23	23	21	23	24	23	21	22	19	19	24	5
W	1	1	1	5	3	3	4	2	1	2	1	5	4
X	6	3	11	4	7	2	6	6	7	7	2	11	9
Y	4	6	7	2	10	5	7	3	5	4	2	10	8

TABLE X

SUMMARY OF THE GRADES GIVEN TWENTY-FIVE ENGLISH PAPERS BY TEN TEACHERS OF ENGLISH

Paper	TEACHERS										RANGE		
	I	II	III	IV	V	VI	VII	VIII	IX	X	High	Low	Diff
A	56	70	77	76	73	75	74	65	51	67	77	51	26
B	75	79	86	82	76	73	69	78	72	69	86	69	17
C	56	68	71	64	65	63	66	60	64	59	71	56	15
D	63	70	78	71	67	69	71	66	72	68	78	63	15
E	81	84	87	84	84	82	75	83	79	85	87	75	12
F	81	84	82	80	82	80	90	80	80	85	90	80	10
G	83	86	89	91	82	85	94	88	86	87	94	82	12
H	74	74	77	76	73	74	76	75	65	74	77	65	12
I	50	58	59	63	50	54	55	68	56	67	68	50	18
J	81	81	90	82	81	78	73	89	88	84	90	73	17
K	88	86	91	88	91	87	88	82	90	90	91	82	9
L	68	75	74	70	80	74	73	56	70	68	80	56	24
M	75	76	78	70	71	77	65	79	76	77	79	65	14
N	63	70	71	65	66	62	63	73	61	65	73	61	12
O	75	75	82	71	75	78	72	64	70	76	82	64	18
P	50	78	69	56	51	66	52	53	51	53	78	50	28
Q	94	94	93	96	92	92	92	95	94	94	96	92	4
R	88	80	89	80	81	83	78	84	81	91	91	78	13
S	85	76	71	71	78	72	62	78	70	56	85	56	29
T	74	80	81	82	86	82	77	76	73	86	86	73	13
U	79	84	88	83	83	81	74	81	72	84	88	72	16
V	63	68	70	67	59	60	61	65	60	68	70	59	11
W	96	95	94	90	88	80	89	93	95	92	96	80	16
X	85	89	86	90	83	90	86	87	81	86	90	81	9
Y	88	85	88	94	82	87	80	91	86	91	94	80	14

TABLE XI

SUMMARY OF THE RANK CORRELATION COEFFICIENTS OF TEN TEACHERS ON TWENTY-FIVE ENGLISH PAPERS

Teacher	I	II	III	IV	V	VI	VII	VIII	IX	X
I	---	.890	.860	.851	.832	.858	.739	.882	.905	.875
II	.890	---	.864	.892	.885	.914	.800	.841	.869	.835
III	.860	.864	---	.892	.842	.883	.803	.857	.922	.898
IV	.851	.892	.892	---	.886	.911	.855	.879	.856	.856
V	.832	.885	.842	.886	---	.903	.841	.766	.835	.826
VI	.858	.914	.883	.911	.903	---	.895	.816	.871	.925
VII	.739	.800	.803	.855	.841	.895	---	.720	.783	.874
VIII	.882	.841	.857	.879	.766	.816	.720	---	.907	.841
IX	.905	.869	.922	.856	.835	.871	.783	.907	---	.903
X	.875	.835	.898	.856	.826	.925	.874	.841	.903	---

As stated in the introduction, this is but a preliminary report on the findings to date in connection with the present study. While the writer does not deem it advisable to draw any conclusions further than those thus far given, it is hoped that the future may bring a greater abundance of conclusive evidence.

Algebra Questions (One Hour)

1. Reduce to lowest terms:

$$\frac{x^2+x-6}{x^2-2x-15}$$

2. Reduce to a fraction:

$$x-3-\frac{2x+1}{x-2}$$

3. (a) Define an algebraic fraction.
   (b) Give the rule for adding fractions in algebra.

4. Reduce the following fractions to equivalent fractions having common denominators:

$$\frac{x-2}{x} \quad ; \quad \frac{2}{x+2}$$

Answers may be left in factored form.

5. Perform the indicated operations:

$$\frac{x}{x-2} - \frac{x-2}{x+2} + \frac{3}{4-x^2} =$$

(The answer may be left in factored form.)

6. Perform the indicated operation:

$$\frac{a^2+3a-18}{a^4-8a^3+12a^2} \cdot \frac{2a^3-4a^2}{a^2-36}$$

7. Perform the indicated operation:

$$\left(\frac{x}{1}+\frac{1}{1}-\frac{20}{x}\right) \div \left(\frac{x}{1}-\frac{2}{1}-\frac{8}{x}\right)$$

8. Explain why in the division of fractions you invert the divisor and multiply.

9. Solve for x in the following equation:

$$\frac{3x-1}{4} - \frac{4x-5}{5} = 4 + \frac{7x+5}{10}$$

Examination Questions for English I

(Composition)

Dictate the first question; write the others on the board.

I. Punctuate the following sentences:
  1. Tom, bring me a drink.
  2. "Mary," she said, "you may leave the room."
  3. He played football in the fall, basketball in the winter, and baseball in the spring.
  4. Mr. Lewis, our mayor, has just returned from New York.
  5. "Treasure Island," which was written by Stevenson, is a good boy's story.
  6. It was a rainy day, but we went just the same.
  7. Boys who sing should join the glee club.
  8. The house was in disorder; things had been thrown everywhere.
  9. In the first place, Elizabeth was too sick to go.
  10. Throwing down his books, John hastily left the room.

II. Write a business letter, renewing your subscription to a magazine.

III. Tell whether the following sentences are simple, complex, or compound. You need not copy the sentence.

1. John and Mary were late.
2. Jane earned an A because she studied her lessons.
3. The storm broke; the rain fell in torrents.
4. Tom, hurrying to lunch, stumbled and fell.
5. When the bell rings you may go.

IV. Name the case of the italicized words. You need not copy the sentence.

1. Jack is my *brother*.
2. They told *Alice* to go.
3. *Whom* are you going to invite?
4. He left it to them, *Sam and Henry*, to decide.
5. The dog broke *its* leg.

V. Write the correct possessive plural forms of the words in parenthesis. Do not copy the sentence.

1. (Lady) hats are expensive.
2. The (child) toys are broken.
3. (Fox) pelts are valuable.
4. The (Green) house was burned.
5. They stole the (Hero) medals.

VI. Write the correct word in the parenthesis in the following sentences. Do not copy the sentence.

1. Has everybody finished (his, their) exercise?
2. I don't like (those, that) kind of people.
3. He inquired (who, whom) I wished to see.
4. Let each speak in (his, their) turn.
5. Neither men nor ships (was, were) lacking.
6. The number of books (are, is) too great for that shelf.
7. I knew it to be (she, her).
8. (Are, is) there any others who wish to go?
9. He treated me (like, as) a criminal.
10. Give it to Harold and (I, me).

VII. (a) Fill in the correct form of the verb in the following sentences. Do not copy the sentence.

1. He (lie) where he fell.
2. How long have you been (sit) here?
3. Where have you (lay) my book?
4. What have you (do) with it?
5. Yesterday I (go) for a walk.

(b) Supply *shall* or *will* in the following sentences. Do not copy the sentence.

1. I.....................ask her for her book tomorrow.
2. I know that he.....................go with her.
3. I.....................do it in spite of her.
4. You.....................report at once.
5. They.....................never know why we came.

VIII. Write a theme of not less than 75 words telling about the last good time that you had.

NEWS NOTES

W. D. Reeve of the University of Minnesota has become a member of the staff of the mathematics department of Teachers College.

Miss Bessie I. Miller, of Rockford College, has published the lectures of her "Browse" Course in Mathematics in a little volume, *Romance in Science*. The chapter titles are Browse, The Scientific Method, Law, The Fourth Dimension, The Fourth Dimension and Non-Euchdean Geometry, Einstein's Theory of Gravitation, Transformations, and the Human Significance of Mathematics.

Professor Clifford Woody has published duplicate forms of his standardized tests in arithmetic. They are distributed by the Bureau of Publications, Teachers College.

The second volume of David Eugene Smith's History of Mathematics has recently been published by Ginn & Company.

Professors H. L. Rietz and A. R. Crathorne have recently published by Henry Holt and Company a text in *Introductory College Albegra.*

Professor Raleigh Schorling is principal of the new University of Michigan High School that is now being opened at Ann Arbor.

At the May meeting of the New York Section of the Association of Teachers of Mathematics in the Middle States and Maryland the program was a conference on "What Mathematics Should Be Taught in the First Year of the New York City High Schools." John A. Swenson of the Wadleigh High School, Nathan Dickler of the Brooklyn Manual Training High School, William S. Schlauch of the High School of Commerce, and Professor Clifford B. Upton of Teachers College were the principal contributors. The chairman was unable to prevail upon any one to defend a year of straight algebra for the first year of the high school course.

Professor J. Andrew Drushel of Harris Teachers College, St. Louis, has accepted a professorship of the teaching of mathematics in New York University.

"A NEW edition of the final report of the National Committee on Mathematical Requirements, entitled 'The Reorganization of Mathematics in Secondary Education' has been printed and is ready for distribution. A nominal charge of twenty cents per copy has been found necessary in order to defray packing and shipping costs. Orders for this report with the necessary remittance may be sent direct to the Dartmouth Press, Hanover, New Hampshire."

RECEIPT of applications for assistant mathematician (tidal) will close November 11, 1924. The examination is to fill vacancies in the Coast and Geodetic Survey, at an entrance salary of $2,400 a year. Advancement in pay may be made without change in assignment up to $3,000 a year. From this position promotions are made successively to the higher grades as vacancies occur at salaries ranging from $3,000 to $5,000 a year.

Applicants must have been graduated with a bachelor's degree from a college of recognized standing with courses in mathematics, including trigonometry, analytic geometry, calculus, analytical mechanics, physics and modern languages. In addition, they must have completed sufficient graduate work for a Master's degree, majoring in mathematics or mathematical physics, or have had at least two years' experience of such a nature as to give a full equivalent of the graduate work called for. . . .

WILLIAM A. LUBY, after a year of graduate study in the department of mathematics at the University of Chicago, assumes his duties as head of the mathematical department in the Kansas City Junior College.

PROFESSOR EARL R. HEDRICK has been appointed head of the department of mathematics in the southern branch of the University of California.

PROFESSOR DAVID EUGENE SMITH has returned to Teachers College after an extended visit in Paris.

PROFESSOR JOHN W. YOUNG, Chairman of the National Committee on Mathematical requirements, is editing the mathematical publications of Houghton-Mifflin Company.